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Research Paper

Mathematical model for phytoplankton growth

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Abstract: Phytoplanktons are producers in aquatic ecosystem and are sources of energy to higher levels (consumers) of food chains and food webs. A simple mathematical model is described in the present paper called nutrient – phytoplankton model. This model tries to understand the dynamics of nutrient driven phytoplankton growth. In the literature surveyed so far, a large number of complicated simulation studies are reported. It is found in actual (realistic) field studies that the growth of the Phytoplanktons is strongly controlled by the nutrients rather than by consumer of higher trophic level (e.g. population densities). The present model is based on the assumption that the growth is accelerated when nutrients exceed a certain level (ecologically saying, that is, oligotrophic water resource turn into eutrophic one). This threshold effect should be generic to simulation models.

Keywords: Nutrients, nutrient – phytoplankton model, phytoplanktons, lentic water bodies.

INTRODUCTION

Phytoplanktons are photosynthetic, tiny plants floating in the surface water in the lentic (non-flowing, stagnant) water bodies, rivers (lotic resources) and oceans. Phytoplankton community occupies the lowermost and starting part of almost all food chains (detritus food chains are exception) and thus is very important functionally. Most of the growth studies of planktons are performed in laboratories and using variety of methods and culture techniques. Therefore, formulation of a mathematical model to understand and explain the phytoplankton growth is difficult. Secondly, most of the experiments are performed in steady-state conditions. A mathematical model, in such situation, can provide the means for keeping track of the complexities and for generalizing the results obtained to transient, non-steady-state conditions. Natural ecosystems contrary to the laboratory culture tanks are characterized by non-steady conditions.
Mathematical models of various aspects of phytoplankton dynamics were formulated earlier. In the literature, the earliest attempt to develop a model of phytoplankton

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productivity was the work of Fleming (1939) which was based on Volterra equations. Sverdrup et al. (1942) and Riley (1946) included light intensity effects on production as well as factors such as temperature – dependent respiration and nutrition depletion. The physical processes such as sinking rate, diffusivity and vertical instability were included in later models (Steele 1958, 1962 and Patten 1963, 1965, 1966). Patten (1968) reviewed early models extensively.

The most commonly used relationship between phytoplankton growth concentrations originates with works of Caperon (1967) and of Dugdale (1967), which used Michaelis-Menten kinetics to describe nitrogen uptake rate by algae as a function of ambient nutrient concentration. Droop (1973) has argued that growth actually responds to size of internal nitrogen pool rather than directly to the external nutrient concentration. Steel and Henderson (1992) have developed model formulations in a simple nitrogen-phytoplankton-zooplankton (NPZ) system that focused on primarily grazing. Fashman et al. (1990) published a model (FDM) which is a well documented and well thought-out model. In this, authors compared its results with field observations. This approach made the basis for present paper.

The growth process and its equation: HE The phytoplanktons grow by absorbing and utilizing the nutrients (mainly nitrogen in the form of nitrite, nitrate and ammonium and phosphate) through a complex metabolism in their cells and tissues. This can be expressed in the form of following equation:-

Uptake
Metabolism

Nutrients ⟶ Absorption and utilization
Growth
(Nitrite, nitrate, ammonium and phosphate)
(Increase in Number/biomass)

THE MODEL

The two nutrient parts for nitrogen and phosphorous represent the fractions that a cell uses for growth. The phytoplankton part represents the numbers. Each of the parts is expressed in gm per cubic meter and numbers per ml of water respectively. The model can be represented by ---

$$dP/dt = \beta_0 \alpha_1 PN - \beta_1 P - \beta_2 P^2 - \beta_3 PZ \quad (1)$$

N : Nutrients
P : Phytoplanktons
$\beta_0 \alpha_1 PN$: Production of Phytoplanktons limitation of N
$\beta_1 P$: Natural depletion rate of Phytoplanktons
$\beta_2 P^2$: Death of Phytoplanktons
$\beta_3 PZ$: Grazing of Phytoplanktons by zooplanktons

Phytoplankton production is expressed by –

$$prPC = \mu\, f(I)\, f(T)\, f(N,P)\, FC\, rd, \quad (2)$$

where μ – maximum growth rate of phytoplankton, $f(I)$ – dependence on light availability, $f(T)$ – temperature dependence, $f(N,P)$ – nutrient function, FC – correction factor for dark reaction, rd – relative day length. The nutrient function is calculated as

$$f(N,P) = \frac{2}{\frac{1}{NF} + \frac{1}{PF}} \quad (3)$$

Where f(N,P) = nutrient function, NF = nitrogen function, and PF = phosphorus function.

The effect of nutrients on production/growth is demonstrated by Figures 1-3 and tables 1-3. It is evident that there is an increase in phytoplanktons and during the blooms it appears to limit the growth severely. The model for growth was tested on this pond and the results are shown in figure 3.

Conclusion: The predicted phytoplankton numbers are plotted against observed numbers during the time period and the agreement is sufficiently good. Recent advances in phytoplankton growth studies permit realistic mathematical modeling of phytoplankton growth. Nutrient uptake can be represented as separate process. The excretion (of nitrogen) and non-predatory mortality can be modeled as functions of other factors (such as temperature, light and limiting factors).

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Table1. Theoretical No. of phytoplankton in a perennial pond in Kota (Rajasthan) India

SN	Time (Days)	No. of phytoplanktons (thousands per ml)
01	001	000
02	071	013
03	141	036
04	221	150
05	281	050
06	323	025
07	365	010

Figure 1. Diagrammatic representation of theoretical No. of phytoplankton in a perennial pond in Kota (Rajasthan) India.

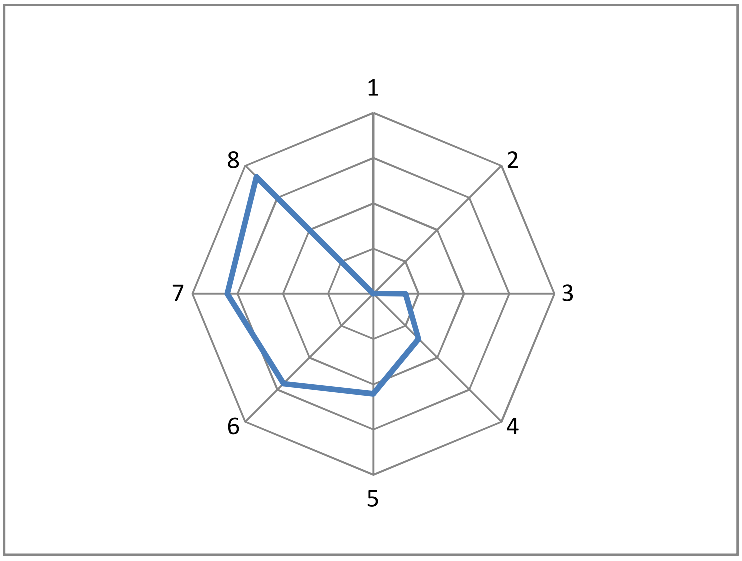

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Table 2. Total count of phytoplankton (Observed data) in a perennial pond in Kota (Rajasthan) India.

Month	No. of Phytoplanktons
01	853
02	1047
03	2471
04	1633
05	2460
06	1829
07	1005
08	2812
09	2646
10	1893

Figure 2. Diagrammatic representation of Total count of phytoplankton (Observed data) of phytoplankton in a perennial pond in Kota (Rajasthan) India.

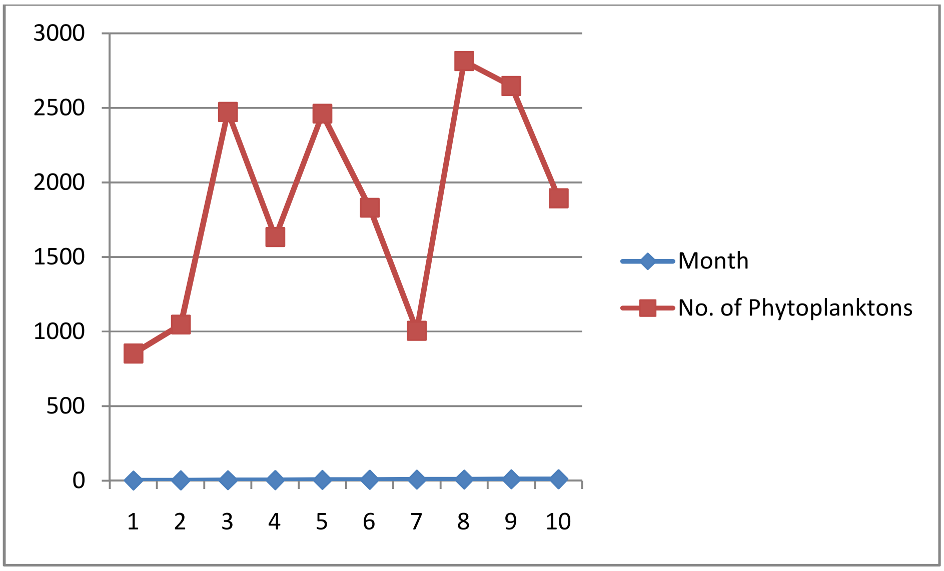

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Figure 3. Diagrammatic representation of comparison of theoretical No. and Total count of phytoplankton (Observed data) a perennial pond in Kota (Rajasthan) India.